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NUMERICAL COMPUTATION OF PARTICLES–TURBULENCE INTERACTION

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Abstract—This paper reviews recent research on the use of numerical simulations to study the motion of particles in turbulent flows. The main emphasis is on direct numerical simulation (DNS). The techniques of DNS are briefly reviewed. The need to establish more appropriate particle equations of motion is discussed and recent progress is summarized. Results for both one-way and two-way (feedback) coupling are reviewed.

Key Words: aerosol, numerical, particle, simulation, turbulence

1. INTRODUCTION

Since the early 1970s direct numerical simulation (DNS) of turbulent flows has become an increasingly useful research tool. Early studies were primarily limited to simulations in periodic boxes, which were models of isotropic, homogeneous turbulence. Simulations of two-dimensional flows, such as mixing layers, began to appear in the 1970s. In the 1980s, simulations of turbulent channel flow and boundary layers became feasible.

The availability of reliable numerical simulations of turbulent or transitional flows has opened-up a number of possibilities for studying the influence of turbulence on various phenomena. Examples include chemical reactions, flow-induced vibrations, heat transfer and particle motion. This review focuses on the latter example. Although studying particle motion with a DNS of turbulence may yield useful insights into turbulence, a primary goal of the research is to gain more insight into the behavior of particles in complex flows. Issues of interest include the diffusion of aerosols from localized sources, the deposition of aerosols on solid surfaces, sedimentation, the feedback of particles on fluid motion and others.

DNS is not the only numerical technique that is being used to study particle motion in turbulent flows. Much understanding has been gained by modeling turbulent flows with random eddies. However, it is not feasible to adequately summarize all the current research being done on particle motion using numerical methods. This review focuses on DNS, since it is the area with which the author has the most experience. However, large eddy simulations (LESs) and stochastic models are likely to be useful for many years, since a DNS is currently feasible only at Reynolds numbers that lie just above the transition point.

Much research has been done on the motion of point particles in turbulence. Examples include Deardorff & Peskin's (1970) pioneering study using Deardorff's (1970) channel flow simulation, as well as recent studies by Bernard *et al.* (1989) and Kontomaris & Hanratty (1993) with DNS of channel flow. However, this paper will focus on "real" particles that possess inertia. Furthermore, most of the discussion will concern aerosol particles, since the particle equation of motion takes a simpler form in this case.

This review places more emphasis on techniques than results. One goal of the paper is to try to provide a summary of the techniques that have been used to date. A second goal is to point out the many difficulties that remain with these techniques. It is hoped that the discussion will stimulate further research on both DNS techniques and particle equations of motion.

2. COMPUTATION OF FLOWS

2.1. DNS of Turbulent Flow

In the late 1960s, it became possible to perform DNS of low Reynolds number turbulent flows in periodic boxes. The Reynolds numbers were restricted to fairly small values because of the limited main memory available on the computers of that era. Orszag & Patterson (1972) reported the results of three such calculations for 32^3 grid points.

Periodic boundary conditions permit the use of Fourier series expansions. In a fully spectral calculation, each term in the Navier–Stokes equation is computed in "spectral space", which means that one works only with the spectral coefficients of the velocity and pressure fields. The disadvantage of a fully spectral DNS is that the nonlinear term in the Navier–Stokes equation involves convolution sums that are extremely time-consuming. It is more efficient to use pseudospectral methods in which the nonlinear terms are computed in physical space. One computes the velocity and spatial derivatives of the velocity in spectral space and then Fourier transforms the results to physical space. While this approach is faster than a spectral evaluation of the nonlinear terms, it introduces aliasing errors.

Orszag (1971a, b) has discussed the effects of aliasing errors on numerical simulations. One may compensate for aliasing by the "two-thirds" rule. A discrete Fourier transform evaluates the N terms in a Fourier series from the values of the function a grid of N equally spaced points. The aliasing error occurs when one multiplies two Fourier series containing N terms. One generates frequencies that cannot be resolved on the grid. Orszag suggested that one should expand the grid to $3N/2$ points to evaluate products such as the nonlinear terms in the Navier–Stokes equation. An equivalent procedure is to set the top third of the Fourier series to zero and then Fourier transform to physical space on a grid containing N points. In discussing simulations, the author will refer to the number of "collocation" grid points on which one evaluates the nonlinear terms. When one uses the two-thirds rule, the effective grid spacing is 1.5 times larger than the grid spacing on the collocation grid, since one cannot resolve length scales corresponding to the Fourier coefficients that vanish.

The DNS of turbulent flows near solid surfaces is considerably more difficult than periodic boxes. Deardorff (1970) performed an early simulation of turbulent channel flow. The simulation was performed by finite-difference methods on a grid with 6720 points. The flow in the core of the channel was treated as inviscid and the law of the wall was used to provide a boundary condition on the velocity field.

Pseudospectral simulations of wall-bounded flows did not appear until the early 1980s. One difficulty associated with rigid boundary conditions is that they induce Gibb's phenomenon when ordinary Fourier series are used to express the dependence of the velocity and pressure fields on the coordinate perpendicular to the wall(s). Gottlieb & Orszag (1977) discuss the problem in depth. Orszag (1971c) recognized that this difficulty could be overcome by using Chebyshev series to expand the fields in the direction normal to the wall(s). However, the actual implementation of Chebyshev expansions was complicated and it was nearly a decade before the first results appeared.

Orszag & Kells (1980) reported the results of a simulation of transitional flow in a channel formed by two infinite, parallel, rigid walls. They studied perturbations of the Poiseuille solution and observed the rapid growth of disturbances in the nonlinear stage of transition. They wrote the Navier–Stokes equation in the rotational form,

$$\frac{\partial \mathbf{u}}{\partial t} = \mathbf{u} \times \omega - \nabla \Pi/\rho + \nu \nabla^2 \mathbf{u}, \qquad [1]$$

since Orszag (1971b) argued that this could improve the stability of a simulation at large Reynolds number. They used a "time-splitting" method to solve the Navier–Stokes equation. In their method, one computes the nonlinear term explicitly using the values of the velocity field on the previous two time steps. One then computes the pressure held by imposing incompressibility on the velocity field. The boundary conditions on the pressure are inviscid. When one imposes the boundary conditions, the resulting Poisson equation for the pressure field is nearly tridiagonal in the Chebyshev index. The computational work involved in solving the Poisson equation is roughly

equivalent to solving a pentadiagonal matrix equation. After adding the pressure field to the velocity field, one must incorporate the viscous term in a third fractional time step. They did this with an Euler backward step that involves solving a nearly tridiagonal matrix for the Chebyshev coefficients.

Orszag & Patera (1981) reported the results of a simulation of channel flow in which they computed the von Kármán constant. However, the flow for which they did the calculation was not in steady state. The spatially averaged wall shear stress had reached a maximum, which was roughly twice the steady-state value, at the time for which they calculated the von Kármán constant.

Moin & Kim (1982) reported the results of 4 LESs of steady-state turbulent channel flow. The channel half-width for their simulation was $h^+ = 320$, which corresponds to a Reynolds number, based on the centerline velocity and the channel half-width, equal to 13,800. In one simulation, they used 65 grid points in the cross-stream direction (y), 64 points in the downstream (x) direction and 128 points in the spanwise (z) direction. The periodicity length in the downstream direction was 8042 wall units and the periodicity length in the spanwise direction was 2011 wall units. They used spectral methods in the x- and z-directions and a finite-difference method in y. To obtain agreement with the experimental fluctuation profile, they used subgrid scale modeling.

Kim *et al.* (1987) reported the first DNS of steady-state turbulent channel flow. They used a fourth-order method that avoids the errors involved in the fractional step methods. They did their calculation for $h^+ = 180$. Their periodicity length in the downstream direction was 2300 wall units, and their periodicity length in the spanwise direction was 1150 wall units. Their spatial grid contained 192 points in the downstream direction, 160 points in the spanwise direction and 129 points in the direction normal to the wall. With the exception of the intensity of the fluctuations in the component of velocity normal to the walls and the skewness and flatness of the normal component of velocity, their results agree with the experimental measurements of Laufer (1954) and Kreplin & Eckelmann (1979). The intensity and flatness and the skewness of the normal component of velocity in the viscous sublayer differ significantly from the experimental observations. Subsequent LDA measurements by Niederschulte (1988) (see also Niederschulte *et al.* 1990) agree better with the results of Kim *et al.* (1987). However, it may be important to test further the validity of the channel flow simulations.

McLaughlin (1989) reported the results of a study of particle motion in a channel flow simulation. Lyons *et al.* (1990) used the same program for a study of coherent structures. The program is a modification of a program developed by Azab & McLaughlin (1987) to simulate the viscous wall region. It is similar to the one developed by Orszag & Kells (1980) with two main differences. Azab & McLaughlin (1987) used a Green's function method devised by Marcus (1984) to include viscous pressure effects. Also, they devised an Adams–Bashforth–Crank–Nicholson (ABCN) fractional step for the nonlinear term that improved the stability of the time-stepping. The ABCN step differs from the one used by Orszag & Kells (1980) in that the spatially averaged velocity profile from the previous time step is used to stabilize the scheme. Orszag & Kells (1980) used the Poiseuille profile in their ABCN step.

McLaughlin's (1989) DNS was for $h^+ = 125$. He used periodicity lengths equal to 630 wall units in both the downstream and spanwise directions. The nonlinear terms were evaluated on a collocation grid with 16 points in the downstream direction, 64 points in the spanwise direction and 65 points in the normal direction. The DNS reported by Lyons *et al.* (1990) was for $h^+ = 150$. The periodicity lengths in the downstream and spanwise directions were 1900 and 950 wall units, respectively. The nonlinear terms were evaluated on a collocation grid with 128 points in the downstream, 128 points in the spanwise and 65 points in the normal direction. Lyons *et al.* (1991) give the details of the channel flow code and a comparison with the results of Kim *et al.* (1987) as well as experimental results.

Another channel flow DNS was recently reported by Rutledge & Sleicher (1993). Their technique is similar to that used by Kim *et al.* (1987), but differs in several details. Their results are for a channel with $h^+ = 180$. The corresponding Reynolds number, based on the bulk velocity and the hydraulic diameter, is 11,200. The periodicity length in the downstream direction was 2262 wall units, and the periodicity length in the spanwise direction was 753 wall units. They made a detailed comparison of their results with those reported by Kim *et al.* (1987) and Lyons *et al.* (1991) as well as the experimental results of Kreplin & Eckelmann (1979), Niederschulte (1989) and Barlow

& Johnson (1985). The three numerical simulations are in close agreement in the viscous wall region. However, there remain unresolved differences with experimental results for higher order moments very close to the wall.

Lam & Banerjee (1992) reported the results of a DNS of open channel flow. They assumed that the free surface was flat but that the tangential stress vanished on the free surface. The dimensionless depth of the liquid was 120·8 wall units (based on the friction velocity at the lower wall where rigid boundary conditions were applied.) Pedinotti *et al.* (1992) used the same DNS in a study of the motion of particle motion near the solid surface. Their results provide insights into the tendency of particles to accumulate in the "low-speed streaks" observed by Runstadler *et al.* (1963) and others.

2.2. Stochastic Methods

Kraichnan (1970) suggested an alternative to DNS for calculating particle trajectories in turbulent flow. He suggested a model for isotropic, homogeneous turbulence based on a spectral representation in the form of a discrete Fourier series:

$$\mathbf{u}(\mathbf{r}, t) = \sum_n \mathbf{u}(\mathbf{k}_n)\exp\left[i(\mathbf{k}_n \cdot \mathbf{r} - \omega_n t\right]. \qquad [2]$$

He picked the wavevectors, $\mathbf{k}_n$, from a statistically homogeneous distribution so that the energy spectrum, $E(k)$, would have the desired shape. For three-dimensional turbulence, he chose the spectrum

$$E(k) = 16\left(\frac{2}{\pi}\right)^{1/2}\frac{v_0^2 k^4}{k_0^5}\exp\left(-\frac{2k^2}{k_0^2}\right). \qquad [3]$$

The frequencies, ω_n, were chosen from a Gaussian distribution with a standard deviation, ω_0. He presented results for $\omega_0 = 0$ and $\omega_0 = k_0 v_0$. For large times, his results are consistent with Taylor's (1921) theory of diffusion of point particles in turbulent flow. Kraichnan applied the direct interaction approximation (DIA) (Kraichnan 1959; Roberts 1961) to point particle diffusion in homogeneous turbulence. He found close agreement between the DIA and the simulations for the dispersion of point particles.

Maxey (1987) used Kraichnan's technique to study the settling of particles in homogeneous turbulence. He also pointed out some deficiencies of the approach. Triple correlations of the fluid velocity vanish and there is no representation of the energy transfer from large to small scales. However, he argued that these were not serious problems if one used the technique to study particle motion in low Reynolds number turbulent flows.

Kallio & Reeks (1989) used a stochastic Lagrangian random-walk numerical technique to provide the fluid velocity at the location of each aerosol in a simulation of wall-bounded turbulent shear flow. This technique has the advantage of being relatively inexpensive, since one does not perform a DNS and one does not need to interpolate the fluid velocity at the location of each aerosol. The latter calculations are the primary expense in simulations involving very large numbers of particles.

3. PARTICLE EQUATION OF MOTION

3.1. One-way Coupling

In the author's opinion, the uncertainties about the appropriate particle equation of motion are usually greater than those associated with the DNS used to generate the velocity field (at least for one-way coupling). Many issues are poorly understood. If one restricts the discussion to "small" particles, one can sometimes make use of results from low Reynolds number hydrodynamics to suggest an appropriate form for the equation of motion. However, even in this limit, there are difficulties.

At a starting point, let us consider the equation of motion derived by Maxey & Riley (1983). These workers considered the motion of a small sphere in a velocity field that varied slowly in space and time. They assumed that one can characterize the velocity field by a length, L, and that $a \ll L$, where a is the radius of the sphere. They also assumed that the characteristic Reynolds numbers

based on the sphere's diameter are small compared to unity. To leading order, the disturbance flow created by the sphere obeys the unsteady Stokes equation. Effects such as the lift force derived by Saffman (1965, 1968) appear at the next order in the particle Reynolds numbers.

An interesting feature of the problem is that the particle's equation of motion does not involve feedback effects. In other words, one needs only to know the undisturbed fluid velocity field to compute how the particle will move. This is a considerable computational advantage in that one can perform the DNS without considering the particle. Unfortunately, although this feature is computationally convenient, it is not satisfied in many situations of practical interest.

The Maxey–Riley theory does not consider interactions between particles. If one wishes to apply their theory to a dispersion of particles in a turbulent flow, the average spacing between particles must be very large compared to their radius. One must also consider the mass loading effect. For example, the density of a dusty gas that is 0.1 volume percent solids can be more than twice as large as the density of the clean gas. This is true because aerosols typically have a density that is $O(10^3)$ times larger than air.

Let us consider the Maxey–Riley theory in more detail. From the unsteady Stokes equation, they showed that

$$\frac{\mathrm{d}\mathbf{v}}{\mathrm{d}t} = \frac{2(\rho-1)}{2\rho+1}\mathbf{g} + \frac{2}{2\rho+1}\left(\frac{\mathrm{D}\mathbf{u}}{\mathrm{D}t} + \frac{1}{2}\frac{\mathrm{d}\mathbf{u}}{\mathrm{d}t} + \frac{a^2}{20}\frac{\mathrm{d}\nabla^2\mathbf{u}}{\mathrm{d}t}\right) - \frac{\dfrac{9}{\sqrt{a^2\,\mathrm{Re}}}}{2\rho+1}\left(\mathbf{v}-\mathbf{u}-\frac{a^2}{6}\nabla^2\mathbf{u}\right) - \frac{\dfrac{9}{a\sqrt{\pi\,\mathrm{Re}}}}{2\rho+1}\int_0^t \frac{\dfrac{\mathrm{d}}{\mathrm{d}\tau}\left(\mathbf{v}-\mathbf{u}-\dfrac{a^2}{6}\nabla^2\mathbf{u}\right)}{(t-\tau)^{1/2}}\,\mathrm{d}\tau. \qquad [4]$$

In [4], $\mathbf{v}$ is the velocity of a spherical particle of radius a. The undisturbed velocity field, evaluated at the position occupied by the center of the sphere, is denoted by $\mathbf{u}$. The dimensionless parameter ρ is the ratio of the particle density to the fluid density. The time derivative $\mathrm{D}/\mathrm{D}t$ denotes a time derivative following a fluid element. The Reynolds number, Re, is based on the characteristic length, L, a characteristic velocity of the undisturbed flow, u_0, and the kinematic viscosity, ν:

$$\mathrm{Re} = \frac{u_0 L}{\nu}. \qquad [5]$$

The terms on the right-hand side of [4] include the effects of gravity-buoyancy, added mass, the pressure gradient in the fluid, Stokes drag and the Basset memory term. The terms involving the Laplacian of the undisturbed fluid velocity represent Faxen corrections.

The Maxey–Riley equation is a generalization of the Basset–Boussinesq–Oseen (BBO) equation for a sphere moving through an otherwise motionless fluid. Basset (1888), Boussinesq (1903) and Oseen (1927) considered a sphere that accelerates from rest in a fluid that is at rest (except for the disturbance created by the sphere). Landau & Lifshitz (1959) present a derivation of the BBO equation.

Much work has been devoted to the motion of aerosols. In this case, the density ratio, ρ, is large compared to unity, and one may neglect many of the terms in [4]. In this case, one may use the following simplified equation of motion:

$$\frac{\mathrm{d}\mathbf{v}}{\mathrm{d}t} = \frac{\mathbf{u}-\mathbf{v}}{\tau} + \mathbf{g}, \qquad [6]$$

where τ is a particle relaxation time that is given by (for $\rho_\mathrm{p} \gg \rho_\mathrm{f}$):

$$\tau = \frac{2}{9}\frac{a^2}{\nu}\frac{\rho_\mathrm{p}}{\rho_\mathrm{f}}. \qquad [7]$$

In [7], ρ_p is the density of the particle and ρ_f is the density of the fluid.

When a particle is close to a solid boundary, [6] should be modified to include the effects of the wall on the drag coefficient. Although Maxey & Riley (1983) did not consider this problem in detail, it appears reasonable to modify the Stokes drag coefficient of the sphere by using Stokes flow results for the drag force on a particle moving through a linear velocity profile near a solid wall. Let us first consider motion perpendicular to the wall. In the following discussion, the mean flow points in the x-direction, the y-direction is perpendicular the wall and the z-direction is the spanwise direction. Cox & Brenner (1967) showed that, for Stokes flow, the drag force on a spherical particle of radius a is given by

$$F_y = 6\pi\mu v_y a \phi_y\left(\frac{y}{a}\right), \tag{8}$$

where the dimensionless quantity ϕ_y may be obtained from an exact series solution derived independently by Brenner (1961) and Maude (1961). The variable y in [8] is the distance of the center of the particle from the wall. The series solution is not convenient for calculations because of its complexity. For large separations, $y \gg a$,

$$\phi_y = -\left(1 + \frac{9}{8}\frac{a}{y}\right) + O\left(\left(\frac{a}{y}\right)^2\right). \tag{9}$$

For small y, Cox & Brenner (1967) used lubrication theory to show that the drag force diverges as the gap between the wall and the particle becomes small:

$$\phi_y = -\frac{1}{\epsilon}\left[1 + \frac{\epsilon}{5}\ln\left(\frac{1}{\epsilon}\right) + 0.971214\epsilon\right], \tag{10}$$

where

$$\epsilon = \frac{y - a}{a}. \tag{11}$$

Dahneke (1974) suggested the following fit to Cox & Brenner's exact result for all particle–wall separations:

$$\phi_y = -\left(1 + \frac{a}{\delta}\right), \tag{12}$$

where

$$\delta = y - a \tag{13}$$

is the gap between the surface of the particle and the wall. Expression [12] differs from the exact result by $< 10\%$.

According to [10], it is impossible for a particle to reach a wall in a finite amount of time. It is the shear stress in the gap that produces the singular behavior in [10]. However, for particles moving through a gas, the continuum approximation breaks down when the gap between the surface of the particle and the wall becomes comparable with the molecular mean-free-path (m.f.p.) of the gas. As pointed out by Cunningham (1910), one effect of this breakdown is that the continuum model overpredicts the magnitude of the shear stress on the surface of the particle in the gap. Dahneke (1974) suggested the following modification of the drag law to incorporate the effects of molecular slip:

$$\phi_y = -\frac{1 + \dfrac{a}{\delta}}{1 + \dfrac{1.4\lambda}{\delta}}, \tag{14}$$

where λ is the molecular m.f.p. Kallio (1989) reported the results of simulations using the drag correction in [14].

The above results for the drag on a particle show that the wall modifies the drag on a spherical particle when the particle is within a few diameters of the wall. When this is the case, the components of the drag force in the horizontal direction are also different. For example, let us consider the x-direction. Let us denote the shear rate by

$$G = \frac{\partial v_x}{\partial y}. \qquad [15]$$

In the viscous sublayer, one might expect the velocity to be predominately in the x-direction. Provided that a particle is very small compared to the thickness of the viscous sublayer, it may be reasonable to use the results of Goldman *et al.* (1967a, b) for the drag force on a sphere near a wall in a linear flow field. They showed that the drag force is given by

$$F_x = 6\pi\mu a v_x \phi_x, \qquad [16]$$

where

$$\phi_x = \phi_{tx} + \frac{a\Omega}{v_x}\phi_{rx} + \frac{yG}{v_x}\phi_{Gx}. \qquad [17]$$

Goldman *et al.* (1967a, b) showed that the functions ϕ_{tx}, ϕ_{rx} and ϕ_{Gx} are functions only of the dimensionless ratio y/a and tabulated them. The quantity Ω is the angular velocity of the sphere. In an unbounded shear flow, $\Omega = -G/2$. However, the presence of the wall reduces the magnitude of Ω. Using the results in Goldman *et al.* (1967a, b), for a torque-free particle,

$$\Omega = \frac{G\left(-\frac{1}{2}t_G - \frac{v_x}{aG}t_t\right)}{t_r}. \qquad [18]$$

Goldman *et al.* showed that t_t, t_r and t_G are functions only of the dimensionless ratio y/a and tabulated them.

Goldman *et al.* (1967a, b) presented asymptotic expressions for the functions in the limit of large values of y/a:

$$\phi_{tx} = -\left(1 + \frac{9}{16}\frac{a}{y}\right), \qquad [19]$$

$$\phi_{rx} = \frac{1}{8}\left(\frac{a}{y}\right)^4, \qquad [20]$$

$$\phi_{Gx} = 1 + \frac{9}{16}\frac{a}{y}, \qquad [21]$$

$$t_t = \left[\frac{3}{32}\left(\frac{a}{y}\right)^2\right]^4, \qquad [22]$$

$$t_r = -\left[1 + \frac{5}{16}\left(\frac{a}{y}\right)^3\right] \qquad [23]$$

and

$$t_G = 1 - \frac{3}{16}\left(\frac{a}{y}\right)^3. \qquad [24]$$

Thus, the effect of the wall on the motion of the particle falls off as a/y at large distances. In this regime, the leading result for ϕ_x is

$$\phi_x = -\left(1 + \frac{9}{16}\frac{a}{y}\right)\left(1 - \frac{Gy}{v_x}\right). \qquad [25]$$

For small values of ϵ, ϕ_x diverges. However, unlike ϕ_y, it diverges only logarithmically. Goldman *et al.* (1967a, b) showed that, as $\epsilon \to 0$,

$$\phi_{tx} = \tfrac{8}{15} \ln(\epsilon), \tag{26}$$

$$\phi_{rx} = -\tfrac{2}{15} \ln(\epsilon), \tag{27}$$

$$\phi_{Gx} = 1.7005, \tag{28}$$

$$t_t = -\tfrac{1}{10} \ln(\epsilon), \tag{29}$$

$$t_r = \tfrac{2}{5} \ln(\epsilon) \tag{30}$$

and

$$t_G = 0.9440. \tag{31}$$

In this limit,

$$\Omega = -\frac{1}{4}\frac{v_x}{a} \tag{32}$$

and

$$\phi_x = \tfrac{8}{15} \ln(\epsilon). \tag{33}$$

In a strongly sheared flow, inertial lift forces are likely to be important. These enter the Maxey–Riley theory at a higher order in the particle Reynolds number than the terms in [4]. Once again, there is a lack of rigorous mathematical justification for using the available results for lift forces. All the existing theories of lift forces at small but finite particle Reynolds numbers are derived for steady (laminar) flows. Thus, one must be guided by physical intuition in treating the lift force. In a turbulent channel or pipe flow, one might expect the lift force on an aerosol to be of greatest importance in the viscous sublayer. In the viscous sublayer, the flow is relatively steady and the strongest mean shear rate occurs in this region. There is unsteadiness in the form of sporadic bursts and sweeps and the flow is three-dimensional. However, the laminar flow results for the lift force have a greater chance of applying in this region than in other parts of the flow.

Saffman (1965, 1968) published a result for the lift force on a small spherical particle in a steady, unbounded, linear shear flow. His result is valid provided that the relevant Reynolds numbers that characterize the disturbance flow created by the sphere are small compared to unity. Two such Reynolds numbers are

$$\mathrm{Re}_G = \frac{|G|d^2}{\nu} \tag{34}$$

and

$$\mathrm{Re}_s = \frac{|v_s|d}{\nu}. \tag{35}$$

In [34] and [35], d is the sphere diameter, G is the shear rate (which is assumed constant), and v_s is the "relative velocity". The relative velocity is the difference between the velocity of the particle and the undisturbed fluid velocity at the center of the sphere. Saffman considered a unidirectional flow with a constant shear rate:

$$\mathbf{u} = Gy\hat{\mathbf{x}}, \tag{36}$$

where $\hat{\mathbf{x}}$ is a unit vector in the x-direction. He also assumed that the particle was constrained to move in the x-direction:

$$\mathbf{v} = \mathbf{u} + v_s\hat{\mathbf{x}}. \tag{37}$$

Saffman assumed that Re_s and Re_G were both small compared to unity. In addition, he considered the strong shear limit defined by

$$\mathrm{Re}_G^{1/2} \gg \mathrm{Re}_s. \qquad [38]$$

He calculated the first two terms in an expansion of the lift force in powers of the particle radius. The first term takes the form

$$\mathbf{F}_l = -6.46\mu v_s a^2 \,\mathrm{sgn}(G)\left(\frac{|G|}{\nu}\right)^{1/2}\hat{\mathbf{y}}, \qquad [39]$$

where sgn denotes the sign of its argument.

Positive values of v_s correspond to particles that are moving faster than the surrounding fluid. In this case, the lift force points in the direction in which the fluid velocity decreases (i.e. the positive y-direction for negative values of G and the negative y-direction for positive values of G).

Saffman's (1965, 1968) result has been generalized in several ways. Harper & Chang (1968) generalized the result to ellipsoidal particles. Drew (1978) considered general two-dimensional linear flow fields. McLaughlin (1991) removed the restriction $\mathrm{Re}_G^{1/2} \gg \mathrm{Re}_s$, and found that

$$\mathbf{F}_l = -\frac{9}{\pi}\mu v_s a^2 \,\mathrm{sgn}(G)\left(\frac{|G|}{\nu}\right)^{1/2} J(\epsilon)\hat{\mathbf{y}}, \qquad [40]$$

where

$$\epsilon = \mathrm{sgn}(Gv_s)\frac{\mathrm{Re}_G^{1/2}}{\mathrm{Re}_s}. \qquad [41]$$

The dimensionless function J is tabulated by McLaughlin (1991). For values of $|\epsilon|$ that are large compared to unity,

$$J = 2.225 - \frac{0.6463}{\epsilon^2}, \qquad [42]$$

while, for $|\epsilon| < 0.25$, J is 2 orders of magnitude smaller than the large $|\epsilon|$ asymptote (2.225). The latter result does not mean that the lift force is reduced by 2 orders of magnitude since Saffman's and McLaughlin's results are only the leading terms in an expansion in the particle radius (or, equivalently, in inverse powers of the viscosity). However, it suggests that, in this regime, the lift force will be much smaller in magnitude than one might guess from an (incorrect) application of Saffman's formula.

Saffman's (1965, 1968) theory does not include the effects of a wall. Drew (1988) and McLaughlin (1993) included the effects of a distant wall. They assumed that $l \gg a$, where l is the distance between the center of the sphere and the wall. It is useful to introduce the dimensionless distance, l_*, that is defined by

$$l_* = \frac{l}{L_G}, \qquad [43]$$

where

$$L_G = \left(\frac{\nu}{|G|}\right)^{1/2}. \qquad [44]$$

For $l_* \gg 1$, McLaughlin showed that the lift force on a particle could be expressed as

$$\mathbf{F}_l = \frac{9}{\pi}\mu v_s a^2\left(\frac{|G|}{\nu}\right)^{1/2}(J^{\mathrm{u}} + J^{\mathrm{w}}), \qquad [45]$$

where J^{u} is given by [40] and J^{w} is a correction due to the wall that is a function of ϵ and l_*. McLaughlin (1993) tabulated J^{w} for values of l_* between 0.1 and 2 and values of $|\epsilon| \geq 0.2$. For values of $|\epsilon| < 0.2$, J^{u} is negligible and the wall contribution is given by Vasseur & Cox (1977). For values of $l_* > 5$, the wall effect may be estimated from the asymptotic result

$$J^{\mathrm{w}} = -\frac{1.879}{l_*^{5/3}}. \qquad [46]$$

McLaughlin's (1993) result provides a connection between Saffman's (1965, 1968) and McLaughlin's (1991) results and theories of Cox & Brenner (1968) and Cox & Hsu (1977). The latter authors derived results for a sphere that is close enough to a solid wall that the wall lies within the "inner" region of the disturbance created by the sphere. This means that one may calculate the leading order lift by regular perturbation methods. On the other hand, they treated the sphere as a point force. Expressed in terms of the notation of this paper, they showed that, to leading order,

$$J = \frac{\pi^2}{16}\left(\frac{1}{\epsilon} + \frac{11}{6} l_*\right). \tag{47}$$

Their results are valid provided that the sphere is many radii from the wall:

$$a \ll l \ll \min(L_G, L_s), \tag{48}$$

where L_s is defined as follows:

$$L_s = \frac{\nu}{|v_s|}. \tag{49}$$

When the Reynolds numbers of the disturbance created by a sphere are small and the sphere translates through an unbounded flow, inertia is a singular perturbation. The lengths in [44] and [49] characterize the distances from the sphere at which inertial effects significantly affect the disturbance created by a sphere translating through a linear shear flow. The length L_G was identified by Saffman (1965) as characterizing the region that is responsible for the leading order lift in the strong shear limit. As pointed out by Vasseur & Cox (1977) and McLaughlin (1993), the linearized form of the Navier–Stokes equation that applies in the "Oseen" region defined by L_G and L_s also applies at distances that are small compared to these lengths but large compared to a. Thus, one can use singular perturbation methods to connect the results obtained by Saffman (1965, 1968) and McLaughlin (1993) with the results derived by Cox & Hsu (1977) for a small sphere in a wall-bounded linear shear flow.

The above results for the lift force are valid provided that $a \ll l$. Leighton & Acrivos (1985) derived the leading-order result for the lift force for a stationary sphere that is in contact with a flat, rigid surface. They assumed that the undisturbed flow is a linear shear flow. The lift force points away from the wall and it varies as the fourth power of the sphere radius. Recently, Cherukat & McLaughlin (1994) have derived results for the range of distances $a < l \ll \min(L_G, L_s)$. Their results can be fitted with the following expression:

$$\mathbf{F}_l = -\rho a^2 v_s^2 I, \tag{50}$$

where the dimensionless factor I is given by

$$\begin{aligned} I = {} & (1.7631 + 0.3561\kappa - 1.1837\kappa^2 + 0.845163\kappa^3) \\ & - \left(\frac{3.24139}{\kappa} + 2.6760 + 0.8248\kappa - 0.4616\kappa^2\right)\Lambda_G \\ & + (1.8081 + 0.879585\kappa - 1.9009\kappa^2 + 0.98149\kappa^3)\Lambda_G^2. \end{aligned} \tag{51}$$

In [51], $\kappa = a/l$ and $\Lambda_G = -Ga/v_s$. The expression for the lift force in [50] and [51] reduces to the Cox–Hsu formula for small values of κ (i.e. $l \gg a$).

The above results are for linear, time-independent velocity profiles. Vasseur & Cox (1977), Cox & Hsu (1977), Schonberg & Hinch (1989) and Drew *et al.* (1991) presented results for time-independent parabolic velocity profiles. However, there are still gaps in our knowledge for parabolic profiles. The author is not aware of any results for more general velocity profiles, for time-dependent flow or accelerating particles.

For particles $< 1\ \mu$m in diameter, Brownian effects are likely to be important. In air, the molecular m.f.p. is roughly $0.1\ \mu$m. There are two effects that should be considered when a particle's size is comparable with or smaller than the molecular m.f.p. The Stokes flow drag coefficients overpredict the actual drag on a particle because of Knudsen-type slip. One way of

modeling this phenomenon is to introduce the slip factor suggested by Cunningham (1910). Davies (1945) offered a modified form of the Cunningham factor:

$$C_c = 1 + \frac{2\lambda}{d}\left[1.257 + 0.4\ \exp\left(-\frac{0.55d}{\lambda}\right)\right]. \quad [52]$$

The second effect that one must consider is Brownian motion. Brownian motion is caused by the impacts of molecules with a particle. If the particle is very large compared to the molecular m.f.p., there are so many collisions within the time needed for the particle to respond that their effects cancel. However, small particles will move in a fashion similar to a random walk.

Gupta & Peters (1985) used a Brownian dynamics (BD) technique to simulate the diffusion of very small particles. The BD technique uses random numbers to model the effects of molecular collisions with small particles. One assumes that a random force acts on the particle. If the random force per unit mass is denoted by **n**,

$$\frac{d\mathbf{v}}{dt} = \frac{\mathbf{u} - \mathbf{v}}{\tau} + \mathbf{g} + \mathbf{n}. \quad [53]$$

One assumes that **n** is a white noise process. The strength of the white noise process may be deduced from the diffusivity of a Brownian particle based on Einstein's theory of Brownian motion (e.g. Becker 1967). Gupta & Peters (1985) discussed a time-stepping technique for solving particle equations of motion similar to [53]. Ounis *et al.* (1991, 1993) used a similar technique to study the motion of submicrometer aerosols in a DNS of turbulent channel flow.

3.2. Two-way Coupling

Even less is known about the equation of motion for two-way coupling problems than for one-way coupling problems. Once again, it appears that the "easiest" case is that of aerosols. One can identify different regimes of particle concentration. For the purpose of discussion, let us take the density ratio, ρ_p/ρ_f, to be 1000. In the truly dilute limit, one can neglect mass-loading effects as well as particle–particle interactions. If one denotes the number of particles per unit volume by n, then the dilute limit corresponds to $na^3 \ll 10^{-3}$. The typical distance between a particle and its nearest neighbor is $l = 1/n^{1/3}$. Thus, in the dilute limit, $l \gg 10a$. In this limit, researchers have generally used [6] or some modification of it.

An intermediate case is when $na^3 \sim 10^{-3}$. In this case, the effective density of the dusty gas is significantly higher than that of the clean gas. Furthermore, if the particle concentration is not uniform, the effective density of the dusty gas will vary in space and time. Tang & Crowe (1989) developed a technique for simulating free shear layers in dusty gases. They treated only two-dimensional, high Reynolds number flows. They solved the following equations:

$$\nabla^2\psi = -\omega, \quad [54]$$

where ψ is the streamfunction and ω is the vorticity; and

$$\frac{D\boldsymbol{\omega}}{Dt} = \frac{1}{\rho_G}\nabla \times \mathbf{f}_D, \quad [55]$$

where f_D is the reaction force of the particles per unit volume on the fluid and ρ_G is the mass density of the clean gas. If the motion is in the x–y plane,

$$\boldsymbol{\omega} = \omega\hat{\mathbf{z}}, \quad [56]$$

where $\hat{\mathbf{z}}$ is a unit vector in the z-direction.

The reaction force in [55] was calculated from

$$f_D = \frac{\rho_p' f(v_p - v_G)}{\tau}, \quad [57]$$

where ρ_p' is the mass of particles per unit volume of the dusty gas and τ is the relaxation time of the particles. The factor f in [57] corrects for finite particle Reynolds numbers:

$$f = 1 + 0.15\,\mathrm{Re}_s^{2/3}. \qquad [58]$$

The particle Reynolds number, Re_s, is defined by [35].

Tang & Crowe (1989) implemented the above equations by seeding the flow with "computational particles". They assumed that each computational particle contained N_p particles all moving with the velocity $\mathbf{v}_p$.

Truesdell & Elghobashi (1991; Elghobashi & Truesdell 1993) used a similar particle reaction force in a DNS of homogeneous turbulent flow. One difference is that they tracked individual particles rather than using computational particles.

4. COMPUTATION OF TRAJECTORIES

In computing trajectories, one may use time-differencing schemes to integrate the ordinary differential equation (ODE) for the particle equation of motion. A variety of schemes have been used in such calculations. As an example, McLaughlin (1989) used a second-order Adams scheme for this purpose.

If one wishes to integrate [6], one needs to evaluate the undisturbed fluid velocity at the point occupied by the center of the particle. Deardorff & Peskin (1970), Riley & Patterson (1974) and Bernard *et al.* (1989) used trilinear interpolation for this purpose. On a marginally resolved grid, there can be significant velocity variations between grid points and it is not clear that linear interpolation will give reliable values. McLaughlin (1989) directly evaluated the spectral sums for the fluid velocity at the location of each particle. This method introduces no errors other than those inherent in the DNS itself. However, it is extremely expensive when large numbers of particles are used.

Yeung & Pope (1988) explored the use of different interpolation techniques to track fluid particles in DNS of homogeneous turbulence. They reported results with 32^3, 64^3 and 128^3 grid points. One conclusion of their study is that linear interpolation gives unacceptable accuracy when it is used to compute Lagrangian statistics. As alternatives to linear interpolation, they studied the use of cubic splines and third-order Taylor series expansions. They found that cubic splines were more accurate than the third-order Taylor expansion. However, cubic splines are also more expensive than Taylor expansions. They found that a 13-point third-order Taylor expansion gave acceptable accuracy in their runs, even though it does not yield continuous approximations.

Balachandar & Maxey (1989) studied the use of direct summation, Lagrangian interpolation, partial Hermite interpolation, linear interpolation and a shape function method in evaluating fluid velocities from Fourier series. They found that partial Hermite interpolation gave acceptable accuracy and was far less time-consuming than direct summation. Partial Hermite interpolation consists of evaluating the exact spectral sum for a velocity component and its spatial derivatives in one coordinate at four points surrounding a point of interest. One then uses Hermite interpolation in the remaining two coordinates to obtain an approximation to the fluid velocity at the point of interest. This process may be efficiently implemented by performing the particle tracking step at a point in the DNS program where the fluid velocity and its spatial derivatives are in physical space in two dimensions and in spectral space in the third direction.

The above procedures may also be used in a DNS of channel flow. In this case, a Chebyshev expansion is used in the direction normal to the wall. Kontomaris *et al.* (1992) studied the use of linear interpolation, third-order Lagrangian interpolation, fifth-order Lagrangian interpolation, cubic splines and partial Hermite interpolation. A conclusion of their study is that, in applications requiring high resolution accuracy on relatively coarse grids, partial Hermite interpolation may be superior to the Lagrangian interpolation schemes. A disadvantage of partial Hermite interpolation is that one must compute the values of several spatial derivatives in addition to the function itself on an array of points. In practice, this means additional CPU time and memory requirements.

Brooke *et al.* (1992) used partial Lagrangian interpolation of order 6 in a study of aerosol motion in a high-resolution channel flow DNS (128 points in x, 65 points in y, and 128 points in z). Ounis *et al.* (1991, 1993) used partial Hermite interpolation in a study of aerosol motion in a lower-resolution channel flow DNS.

5. RESULTS FOR ONE-WAY COUPLING

An early study of particle motion in a decaying turbulent flow by Riley & Patterson (1974) is a typical example of a DNS of turbulence in a periodic box. Their DNS was for a cubic box. They used 32 grid points in each direction. They used a Gaussian random initial condition and time-evolved the flow until the skewness had reached a quasi-steady value and the transfer spectrum developed a shape that is characteristic of turbulent flows. At this point, small particles were seeded in the flow field and their trajectories were computed. They ignored the interactions between particles and the feedback of the particles on the flow. They included only a Stokes drag in the particle equation of motion.

Riley & Patterson (1974) studied Lagrangian and Eulerian two-point correlations. For fluid particles, the Lagrangian autocorrelation was larger than the Eulerian correlation for shorter times, but for larger times the opposite was true. They found that the Lagrangian autocorrelation for real particles increased as the relaxation time, τ, increased.

Since they ignored the effect of gravity, Riley & Patterson did not observe the "crossing trajectories" effect that was identified by Yudine (1959) and discussed by Csanady (1963). The crossing trajectories phenomenon is caused by particles "falling out" of an eddy. Particle inertia can also cause a particle to be "centrifuged" out of an eddy by its own inertia. It is difficult to distinguish between these effects with experimental methods. Snyder & Lumley (1971) reported the results of experiments in which they photographically tracked particles in grid-generated turbulence and discussed the diffusion particles and the effect of the particle relaxation time. Wells & Stock (1983) performed experiments with electrically charged particles. By imposing an electric field, they were able to reduce the effect of gravitational settling. They found that the crossing trajectories effect is not important for particles having settling velocities that are small compared to the r.m.s. fluid velocity. When it is significant, the crossing trajectories effect tends to reduce the diffusion of particles. The reduction of the diffusivity for motion normal to the settling direction is greater than the reduction of the diffusivity parallel to the settling direction.

Squires & Eaton (1991a) have investigated the above phenomena with a DNS of homogeneous turbulence. An advantage of DNS is that one can delete either of the terms in [6]. They found that the inertia effect tends to increase particle diffusion and that the crossing trajectories tend to reduce particle diffusion. Their results agree with the theoretical predictions of Reeks (1977). Yeh & Lei (1991) found consistent behavior with an LES of homogenous turbulence.

Numerical simulations are also being used to study the deposition of aerosols onto solid surfaces. Aerosol deposition is a very complex phenomenon. Gravity, Coulombic and Van der Waals forces, inertia, Brownian motion, wall corrections to the drag laws, the orientation of the surface, particle rebound and the smoothness of the surface are only some of the issues that must be addressed. If one restricts attention to aerosol deposition in vertical channels or pipes, gravity cannot directly cause the deposition of particles. Provided that the aerosols are not charged, one can also eliminate Coulombic forces. For aerosols that are larger than about 1 μm, Brownian motion should be unimportant. However, even with these simplifications, the problem is still difficult.

Friedlander & Johnstone (1957) carried out experiments on the deposition of aluminum and iron dust in a vertical pipe. They used double-sided adhesive tape to trap the particles and then counted them to determine the deposition rate. Based on their findings, they proposed a model of aerosol deposition based on the concept of free-flight. They suggested that aerosols move from the core of the pipe to the edge of the viscous sublayer by turbulent diffusion. They assumed that all aerosols that reach the edge of the sublayer reach the wall by a free-flight. This means that the aerosols have sufficient momentum to reach the wall without any "help" from the fluid, since they assumed that the sublayer was stagnant. Thus, one must assume that the aerosols have normal components of velocity that are very large compared to the intensity of the normal component of the fluid

velocity at $y^+ = 5$. For example, for an aerosol with $\tau^+ = 5$ to reach the wall from $y^+ = 5$, its normal component of velocity must be at least unity in wall variables (i.e. it must be at least as large as the friction velocity.)

Numerous authors criticized the Friedlander–Johnstone (1957) theory. Cleaver & Yates (1975) proposed a different model in which the aerosols are carried to the wall by eddies. Their model makes use of experimental findings by Runstadler *et al.* (1963), Kline *et al.* (1967) and Corino & Brodkey (1969) that the viscous sublayer is not a "laminar sublayer" as people had previously thought. It is now known that the flow in the viscous sublayer is three-dimensional and time-dependent. Flow visualization experiments reveal the presence of "low-speed streaks" that have a typical spanwise spacing equal to 100 wall units. Sporadic sweeps and bursts cause an exchange of fluid and particles between the sublayer and the core of the flow.

McCoy & Hanratty (1977) reviewed a number of experimental studies of aerosol deposition in vertical pipe flows. They suggested a fit to the experimental deposition rates as a function of the dimensionless relaxation time, τ^+. For $\tau^+ > \sim 20$, the deposition rate is roughly constant. However, for $\tau^+ < \sim 20$, the dimensionless deposition rate is proportional to the square of τ^+:

$$k_D^+ = 3.25 \cdot 10^{-4} \tau^{+2}. \qquad [59]$$

The deposition rate is defined by

$$k_D = \frac{j}{c}, \qquad [60]$$

where j is the flux of particles onto the wall(s) and c is the number of particles per unit volume in the core of the duct. Based on their own experiments with olive oil droplets, Liu & Agarwal (1974) also concluded that the dimensionless deposition rate is proportional to $(\tau^+)^2$ for $\tau^+ < 20$. For very small values of τ^+, Brownian motion affects the deposition rate for realistic laboratory flows. It is not possible to give a precise value of τ^+ at which Brownian effects will dominate, since the Brownian diffusivity depends on the physical size of the particle. However, for particles in the Brownian regime, the deposition rate decreases with τ^+ if one fixes the material of the particle (e.g. polystyrene) and the friction velocity and varies the radius of the particles.

Kallio & Reeks (1989) reported a numerical study of aerosol deposition in a turbulent channel flow. They assumed that, when a particle strikes the wall, it adheres to it. Such an assumption may be reasonable for aerosol droplets, provided that they are not too large. They included the Saffman lift force and the Stokes drag force in the particle equation of motion, but they neglected the effect of gravity. Their simulation may be viewed as a simulation of a vertical duct flow. There are indirect ways in which gravity might play a role in particle deposition in vertical ducts. For example, in a vertical flow, a particle's sedimentation velocity will contribute to the lift force acting on the particle. However, estimates of the size of such effects for realistic laboratory flows suggest that they are not important (e.g. McLaughlin 1989).

Kallio & Reeks (1989) performed calculations for values of τ^+ between 0.3 and 1000. They compared their predictions with the experimental findings of Liu & Agarwal (1974). For $\tau^+ > 10$, the experiments and numerical results are in agreement. For smaller values of τ^+, simulations which include the Saffman lift force overpredict the experimental values for $\tau^+ > \sim 3$ and underpredict the experimental values for smaller values of τ^+. When the Saffman lift force is not included, the simulations underpredict the experimental observations for $\tau^+ < \sim 5$. The Saffman lift force consistently increases the computed deposition rates for $\tau^+ < \sim 20$.

Kallio & Reeks (1989) also studied the time evolution of the particle concentration in their simulation. Although the particles were initially seeded uniformly in the flow, they observed an increase in the particle concentration in the viscous sublayer for particles with too little inertia to be projected across the sublayer. This phenomenon is similar to "turbophoresis" which was discovered by Caporaloni *et al.* (1975) and, independently, by Reeks (1983).

McLaughlin (1989) performed a numerical study of aerosol deposition in a vertical channel using DNS to provide the fluid velocity. He studied aerosols in the range $2 \leqslant \tau^+ \leqslant 6$ and with the density ratio corresponding to olive oil droplets in air. In some calculations, he included the Stokes drag force and the Saffman lift force in the aerosol equation of motion. In other calculations, he included only the Stokes drag force. When the Saffman lift force was not included, his simulations

underpredicted the experimental values over the entire range. When the Saffman force was included, the predicted deposition rates were higher than the experimental values for $\tau^+ > \sim 1.5$. Although the Saffman lift force had a significant effect on the deposition rate, McLaughlin also noted that the conditions for the validity of the Saffman theory were not satisfied. Particles that deposit typically move through the viscous sublayer with relatively large normal components of velocity. As a result, the particles develop large streamwise relative velocities. The particle Reynolds numbers typically reach values of order unity. The principal contribution to the Reynolds number is from the streamwise component of velocity. In addition, the ratio ϵ defined in [41] is typically smaller in magnitude than unity. This is also a consequence of the large streamwise relative velocity that results from the particle's large normal component of velocity and the steep fluid velocity gradient in the viscous sublayer. Saffman (1965, 1968) assumed that the particle Reynolds numbers were small compared to unity and that $|\epsilon|$ was large compared to unity. McLaughlin's (1991) generalization of Saffman's theory suggests that the true shear-induced lift force is much smaller than Saffman's formula would predict for $|\epsilon| < 1$. The asymptotic theory agrees with the experimental observations of particle migration velocities in a plane Couette device by Cherukat *et al.* (1994) for $\mathrm{Re}_s < 1$.

Like Kallio & Reeks (1989), McLaughlin (1991) noted a tendency of particles to accumulate in the viscous sublayer. This phenomenon is caused by the variations in the intensity of the normal component of the fluid velocity. Particles are thrown toward the wall by strong fluid motions. If the particles do not have enough momentum to reach the wall, they can reside for very large amounts of time near the wall, where the intensity of the normal component of fluid velocity is very small. While this is similar to turbophoresis, it is not clear that it can be described by the same mathematics. Turbophoresis is a drift motion that is superposed on an otherwise random walk. The aerosol almost follows the fluid, but it overshoots the motion of a fluid particle because of its own inertia. These overshoots result in a drift because of the gradient in the intensity of the normal component of the velocity. However, McLaughlin observed aerosols that were projected toward the wall with very large normal components of velocity [comparable with the values of the core of the flow, as suggested by Friedlander and Johnstone (1957)] and which move in a unidirectional fashion until they reach a deep point within the viscous sublayer.

McLaughlin also evaluated the Reynolds number of the aerosols that deposit as they move toward the wall. Even for $\tau^+ = 2$, aerosols develop Reynolds numbers that are close to unity. To quantify this result, he performed conditional averages of the aerosol Reynolds number at various distances from the wall. The condition is that an aerosol is on a unidirectional flight to the wall at a given point. For $\tau^+ = 2$, the conditionally averaged Reynolds number is 0.7 at $y^+ = 2$. If one computes the average Reynolds number of all aerosols at a given y^+ it is much smaller than the conditionally averaged value. The largest value of the average Reynolds number for $\tau^+ = 2$ is 0.03.

Thus, there are some aerosols for which the use of low Reynolds number assumptions is questionable. An *ad hoc* procedure is to use a correlation such as [58] to correct the drag coefficient. Clift *et al.* (1978) have compiled a number of such correlations and their ranges of applicability. As long as wall effects and/or velocity gradient affects are not important, this procedure is reasonable since one is using the correlation for conditions similar to those of the experiments that were used to generate the correlations. However, when a particle is close to a wall or when the flow is strongly sheared, it is not clear that one can obtain reasonable results by superposing results for the lift force with the drag force that is based on measurements far from the nearest wall.

Although this review emphasizes macroscopic particles, some related research has been done on macromolecules by Massah *et al.* (1993). Massah *et al.* (1993) used a DNS of turbulent channel flow to study the motion of high molecular weight polymers in turbulent shear flows. Toms (1948) found that very small amounts of high molecular weight polymers can significantly reduce turbulent drag. Lumley (1967) suggested that the polymers unravel when exposed to high strain rates and selectively increase the local rate of dissipation of turbulence energy. It is difficult to test this idea in the laboratory. Using a bead–spring model of a polymer molecule, Massah *et al.* (1993) studied the conformation of a random polymer chain both in laminar shear flows and a DNS of turbulent channel flow. They found that the chains unravel even in simple shear flows for strong shear rates.

Surprisingly, they found that the chains tend to align with the flow direction rather than the strain axis. In a turbulent shear flow, the polymers unravel in the viscous sublayer and make a 7° angle with the mean flow direction. However, to date, no results for polymer feedback have been published.

6. RESULTS FOR TWO-WAY COUPLING

There are fewer numerical simulation results for two-way coupling than for one-way coupling. There is a much larger body of experimental research on turbulence modification by particles. Hetsroni (1989) has reviewed research on two-way coupling in turbulent flow and the reader is referred to that paper for a discussion of experimental work in the field.

The two-dimensional shear layers considered by Tang & Crowe (1989) are not turbulent, but they involve vortical motion and the authors were able to check their findings against experimental observations. The large vortical motions in the shear layer tend to centrifuge particles from the interiors of the vortices. This phenomenon is similar to that predicted by Maxey (1987) in his study of the sedimentation of particles through homogeneous turbulence. Tang & Crowe (1989) illustrate the phenomenon with a photograph of a particle-laden mixing layer taken from an experimental study by Kamalu *et al.* (1988).

Tang & Crowe (1989) found that the particles inhibit the pairing of vortices. They seeded the shear flow with a uniform distribution of particles at the beginning of their numerical experiments. Initially, the flow is a perturbation of a one-dimensional free shear layer. The elementary vortices used in their numerical technique are arranged to form four identical sinusoidal rows. Each row consists of two sinusoidal waves in wavelength λ. If the velocities of the free streams are denoted by U_1 and U_2, one can define a velocity scale, U, by

$$U = \frac{U_1 - U_2}{2}. \qquad [61]$$

One may introduce a "Stokes number", St, that is a measure of particle inertia as follows:

$$\mathrm{St} = \frac{\tau U}{\lambda}. \qquad [62]$$

The Stokes number may be viewed as a dimensionless relaxation time. For turbulent flow, the friction velocity, u_*, and the kinematic viscosity are used to nondimensionalize the relaxation time. Tang & Crowe (1989) performed calculations for $\mathrm{St} = 10$.

Tang & Crowe (1989) characterized the concentration of particles by the mass concentration ratio, C,

$$C = \frac{\rho_p'}{\rho_G} \qquad [63]$$

(see [57]). They reported results for $C = 10$.

When they ignored the feedback of the particles on the gas, Tang & Crowe (1989) found that neighboring vortices merged in a time equal to $2.36\lambda/U$. However, when they included particle feedback, they found that the vortices did not merge until $4.20\lambda/U$. The final structures are similar in size and appearance, and the particles are centrifuged out of the vortex cores in both cases. Thus, the particles slow the vortex-pairing process.

Two numerical simulations of particle feedback on turbulence have appeared in recent years. Squires & Eaton (1990) studied particle motion in a steady-state homogeneous turbulent flow. Elghobashi & Truesdell (1993) studied particle motion in a decaying (in time) homogeneous turbulent flow. An advantage of homogeneous turbulence is that the flow is less complicated than shear flows and this may simplify the interpretation of the results.

To obtain steady-state turbulence, one must supply energy to the flow. Squires & Eaton (1990) choose to feed energy to the low-wavenumber component of their flow. They used [6] without the gravity term. They investigated a range of solid volume fractions for which significant feedback effects occur. However, in all cases, the volume fractions were small enough that one could ignore

particle–particle interactions. They used simulations with 32^3 and 64^3 points. In some simulations, they used $3.73 \cdot 10^5$ particles, while, in others, they used 10^6 particles. They varied the mass loading from 0.1 to 1. In each simulation, they allowed no coupling between the phases until a statistical steady state had been reached.

When no feedback was allowed, Squires & Eaton (1990) found a very pronounced tendency for particles to accumulate in regions of low vorticity. This behavior is consistent with Maxey's (1987) predictions. They found that this tendency was less pronounced for particles having larger Stokes numbers (i.e. particles that have more inertia).

When Squires & Eaton (1990) allowed feedback, they found that the turbulence energy decreased and the rate of dissipation increased by as much as a factor 2. When they investigated the wavenumber spectra, they found that the energy in the highest wavenumbers increased relative to the energy in the lower wavenumbers because of the particles.

Squires & Eaton (1990) also found that feedback modifies the tendency of particles to accumulate in regions of low vorticity. They found that particles having a Stokes number that is small compared to unity are less likely to accumulate in regions of low vorticity. However, for particles with Stokes numbers closer to unity, they observed the opposite effect. However, for both types of particles, as one increases the mass loading, there is an increased tendency for particles to accumulate in regions of high strain rate and low vorticity.

A goal of the calculations by Elghobashi & Truesdell (1993) was to identify the influence of particles on the cascade of turbulent energy through wavenumber space. Thus, they chose to investigate decaying turbulence to simplify the analysis of the energy transfer. Although they performed calculations for aerosols, they retained simplified forms of the added mass, pressure gradient and Basset terms in addition to the Stokes drag and gravity-buoyancy terms. As expected the Stokes drag and gravity-buoyancy terms were dominant.

Elghobashi & Truesdell (1993) investigated the effects of particle response time, particle diameter, particle volume fraction and gravity on turbulence structure. Quantities of interest included the total turbulent energy, the dissipation rate, the spectral decomposition of these quantities and the rate of energy transfer.

To generate homogeneous turbulence, Elghobashi & Truesdell (1993) initialized the Fourier coefficients of the velocity field according to the energy spectrum

$$E(k) = \frac{3u_0^2}{2} \frac{1}{2\pi} \frac{k}{k_p^2} \exp\left(-\frac{k}{k_p}\right). \qquad [64]$$

They time evolved the velocity field until the energy cascade in spectral space has been established. They then introduced particles into the flow. Each particle was given a velocity equal to the undisturbed fluid velocity at the location of the particle.

To investigate the effect of the particle relaxation time, they fixed the particle diameter and varied the density of the particle. They found that the rate of dissipation increased with τ. To gain more insight into this phenomenon, they studied the wavenumber spectra of the energy and the dissipation rate. For dimensionless wavenumbers < 18, the particles increase the energy spectrum. For larger wavenumbers, the particles lower the energy spectrum. Associated with this behavior is a corresponding increase or decrease in the spectrum of the dissipation rate. Thus, the particles tend to redistribute energy from high wavenumbers to low wavenumbers. When they studied the spectrum of the rate of energy transfer, $T(k)$, they found that the particles tend to suppress the transfer of energy for dimensionless wavenumbers < 18. The particles have the opposite effect for larger wavenumbers.

The particles in Elghobashi & Truesdell's (1993) DNS were smaller than the Kolmogorov scale of the turbulence. Thus, one would not expect them to directly influence small wavenumbers (large lengths). The physical explanation for their observations is that the particles impart their energy to small eddies (high wavenumbers). This increases the rate of dissipation at high wavenumbers. The transfer function is affected by nonlocal triad interactions in Fourier space. When the dissipation rate is increased at large wavenumbers, the transfer function at smaller wavenumbers is increased. Thus, the energy of the smaller scales in the energy-containing eddies is more rapidly depleted. Since these smaller eddies are suppressed, the energy in the largest eddies tends to reside in them for a larger time.

To investigate the effect of particle diameter, Elghobashi & Truesdell (1993) fixed the particle relaxation time and varied the particle material density, ρ_p. In doing this, they held the volume fraction and the relaxation time of the particles constant. Thus, by decreasing ρ_p, they decreased the particle mass loading and the number of particles per unit volume. The effect of varying the particle diameter on the turbulent energy and rate of dissipation was considerably smaller than the effect of varying the relaxation time for a fixed diameter.

Elghobashi & Truesdell (1993) also investigated the effect of varying the volume fraction of particles for a fixed type of particle. As expected, they found that the effects on the energy and rate of dissipation increased with the volume fraction.

In performing the above calculations, Elghobashi & Truesdell (1993) neglected the effects of gravity. They also investigated the effects of gravity. In principle, one could compare the results of these computations with the experiments performed by Wells & Stock (1983). The feedback of the particles makes the flow anisotropic. Qualitatively, the effect of the particles on the energy and dissipation spectra is similar to the gravity-free case. However, as gravity is increased, the "cross-over" wavenumber decreases from 18 to 9 for the case in which gravity is strongest. Further, the cross-over wavenumber decreases with time. The pressure–strain correlation redistributes energy from the vertical direction to the two horizontal energies and a reverse cascade energy from small scales to large scales results. This reduces the rate of decay of turbulence energy compared to the gravity-free case.

7. CONCLUSION

Given the unresolved questions surrounding DNS techniques as well as the particle equation of motion, it is probably true that the main value of the research in this field has been to stimulate experimental and theoretical research on relevant issues. Perhaps one can view many of the results to date as a sensitivity analysis in which one selectively includes or deletes various effects such as gravity, particle inertia, lift forces and others. This approach permits one to obtain a rough estimate of the importance of such effects in various phenomena such as particle dispersion.

By far the biggest challenge is to develop techniques to simulate particle-laden flows when the concentration of particles is enough to significantly modify the flow. Some steps have been taken in this direction. However, the inclusion of effects such as wall corrections to the drag, lift forces and effects like the Basset memory term that may be important for liquid flows will require further modeling and theoretical developments. Even in the dilute limit, there is much theoretical work that, in the author's opinion, should be done to put particle calculations on a sounder footing.

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